



## Functions as Parameters

- Functions as Parameters
- Map sq Function
- Parameter Types
- Generic Parameters
- Mismatched types
- Explicit Generics
- Generics
- Exercise
- Class vs. Type
- Binary Mapping
- Filtering
- FoldBack
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- More Generic FoldBack
- Closures
- Local functions
- Staged computation

# Functions as Parameters

# Functions as Parameters

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## Example (C# function passing)

```
delegate int func(int x);
int comp(func f, int z) { return f(z); }
int sq(int x) { return x*x; }

static void Main(){ comp(sq, 4); } // return 16
```

- A function can be executed by another function
- Functions can be passed as parameters

# F# Functions

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## Example

```
> let comp f z = f z;;  
val comp : f:(('a -> 'b) -> z:'a -> 'b)  
  
> comp sq 4;;  
val it : int = 16
```

- Functions can be passed as parameters
- Parameter function executed by another function
- Supports abstraction such as repeated function execution

# Mapping sq Function

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## Example

```
> let sq x = x*x;;
> let rec map_sq L =
  match L with
  | [] -> []
  | h::t -> sq(h)::map_sq t;;

> map_sq [1;2;3;4];;
val it : int list = [1; 4; 9; 16]
```

- Repeated mapping of unary sq function to every element of one list:  $\text{sq}(1) :: \text{sq}(2) :: \text{sq}(3) :: \text{sq}(4) :: []$

# Parameter Types

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## Example

```
> let rec map (f: int -> int) (L: int list) =  
    match L with  
    | [] -> []  
    | h::t -> (f h)::map f t;;  
  
> map sq [1;2;3;4];;  
val it : int list = [1; 4; 9; 16]
```

- map parameter `f: int->int` is function with int parameter and returns an int
- There are no int-specific operations in map, but map is not generic, only allows `int->int`

# Generic Parameters

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## Example

```
> let sq (x:int) : int = x * x;;
```

```
> let rec map (f: 'a -> 'a) (L : 'a list) =  
  match L with  
  | [] -> []  
  | h::t -> f h::map f t;;
```

```
> map sq [1;2;3;4];;  
val it : int list = [1; 4; 9; 16]
```

- Use 'a when parameter can be any type
- map operations are generic
- But sq uses int-specific operation \*
- Pass type parameter, creating generic map

# Mapping *foo*

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## Example

```
> let foo (x: string) : string = x + "#";;
```

```
> let rec map (f: 'a -> 'a) (L : 'a list) =  
    match L with  
    | [] -> []  
    | h::t -> f h::map f t;;
```

```
> map foo ["a";"b";"c"];;  
val it : string list = ["a#"; "b#"; "c#"]
```

- A *unary* function has a single input parameter
- map of unary *foo* function to every list element

# Mismatched parameter/result types

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## Example

```
> let ItoS (x: int) : string = x.ToString();;
```

```
> let rec map (f: 'a -> 'a) (L : 'a list) =
```

```
    match L with
```

```
    | [] -> []
```

```
    | h::t -> f h::map f t;;
```

```
> map ItoS [1;2;3];;
```

```
error FS0001: Type mismatch. Expecting a int -> int  
but given a int -> string
```

```
The type 'int' does not match the type 'string'
```

- map expects `f : 'a -> 'a` – parameter/result must be same type



# Fixing mismatched types

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## Example

```
> let ItoS (x: int) : string = x.ToString();;
> let rec map<'T1, 'T2> (f:'T1->'T2) (L:'T1 list) =
    match L with
    | [] -> []
    | h::t -> f h::map f t;;

> map ItoS [1;2;3];;
val it : string list = ["1"; "2"; "3"]
```

- First way of solving mismatched types
- Note the use of <...> after the function name

# Fixing mismatched types

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## Example

```
> let ItoS (x: int) : string = x.ToString();;
> let rec map (f : 'a->'b) (L : 'a list) =
    match L with
    | [] -> []
    | h::t -> f h::map f t;;

> map ItoS [1;2;3];;
val it : string list = ["1"; "2"; "3"]
```

- Second way of solving mismatched types

# Exercise

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### Exercise

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## Given

```
let f a = [a];;  
let g a = a / 3.0;;
```

## Results?

```
map f [1; 2; 3];;  
map f [1.0; 2.0; 3.0];;  
  
map g [1; 2; 3];;  
map g [1.0; 2.0; 3.0];;
```

# Class vs. Type

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## From Design Patterns: Elements of Reusable Object-Oriented Software

*An object's class defines how the object is implemented. The class defines the object's internal state and the implementation of its operations.*

*In contrast, an object's type only refers to its interface—the set of requests to which it can respond.*

*An object can have many types, and objects of different classes can have the same type.*

- Consider a *type* Stack with operations: push, pop, isEmpty
- One *class* for String, another *class* for Int

# Binary Mapping

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## Example

```
> let add x y = x + y;;

> let rec map2 f L1 L2 =
    match (L1,L2) with
    | ([], []) -> []
    | (h1::t1, h2::t2) -> f h1 h2::map2 f t1 t2;;

> map2 add [1;2;3] [4;5;6];;
val it : int list = [5; 7; 9]
```

- Repeated mapping of *binary* add function to every element of two lists with equal number elements

# Filtering maps with a guard

## Example

```
> let odd x = x%2=1;;

> let rec filter f L =
  match L with
  | [] -> []
  | h::t when f h -> h::filter f t
  | h::t -> filter f t;;

> filter odd [1;2;3;4;5];;
val it : int list = [1; 3; 5]
```

- Filter a list elements by repeated mapping of odd predicate function to every element of one list

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### Filtering

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# FoldBack

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## Example

```
> let mult a b = a * b;;
> let rec foldb_mult L a =
  match L with
  | [] -> a
  | h::t -> mult h (foldb_mult t a);;
> foldb_mult [2;3;4;5] 1;;
val it : int = 120
```

- Reduction of a list by repeated mapping of binary function from elements of one list to a single combined value
- Equivalent to:  
(mult 2 (mult 3 (mult 4 (mult 5 1))))

# FoldBack

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### Generic FoldBack

More Generic FoldBack  
Closures  
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## Example

```
> let mult a b = a * b;;  
> let rec foldback f L a =  
    match L with  
    | [] -> a  
    | h::t -> f h (foldback f t a);;  
  
> foldback mult [2;3;4;5] 1;;  
val it : int = 120
```

- In this example, function parameters and `a` and result all have same type.



# More Generic FoldBack

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## Example

```
> let cons x L = x::L;;
> let rec foldback f L a =
    match L with
    | [] -> a
    | h::t -> f h (foldback f t a);;

> foldback cons [1;2;3] [];;
val it : int list = [1; 2; 3]
```

- Requires *second* function parameter and reduction value a and result to have same type.

# Closures

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Function and its environment – a table storing a reference to each of the non-local variables of that function.

A closure – unlike a plain function pointer – allows a function to access those non-local variables even when invoked outside of its immediate lexical scope.

# Local functions

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## Example

```
let prime n =  
  let rec remainder m =  
    match m with  
    | 0 | 1 -> true  
    | m ->  
      if n%m=0 then false  
      else remainder (m-1)  
  in remainder (n-1);;
```

- Observe that `n` parameter remains fixed in `remainder`
- Compute `remainder` with one parameter

# Staged computation

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## Example

```
> let rec foldback f L a =  
    let rec red M =  
        match M with  
        | [] -> a  
        | h::t -> f h (red t)  
    in  
    red L;;  
  
> foldback add [4;5;6] 0;;  
val it : int = 15
```

- Improves foldback by eliminating fixed parameters  $f$  and  $a$
- `red L` returns a reference to the function bound with the enclosing environment of  $f$ ,  $L$ , and  $a$ .
- More later when we discuss currying.